Comparison of Weighted Sum Fitness Functions for PSO Optimization of Wideband Medium-gain Antennas

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Abstract. *In recent years PSO (Particle Swarm Optimization) has been successfully applied in antenna design. It is well-known that the fitness function has to be carefully chosen in accordance with the requirements in order to reach an optimal result. In this paper, two different wideband medium-gain arrays are chosen as benchmark structures to test the performance of four PSO fitness functions that can be considered in such a design. The first one is a planar 3 element, the second one a linear 4 element antenna. A MoM (Method of Moments) solver is used in the design. The results clearly show that the fitness functions achieve a similar global best candidate structure. The fitness function based on realized gain however converges slightly faster than the others.*

Keywords

Fitness function, multi-objective optimization, wideband antenna array.

1. Introduction

PSO (Particle Swarm Optimization) has been applied successfully in a growing number of applications involving the design of electromagnetic systems of increasing complexity [1-7]. In order to apply conventional PSO in multi-objective antenna optimization problems, the multiple objectives are normally weighted and combined to form a single objective. This single objective is usually a sum of weighted S_{11} (or VSWR) and gain at all the frequencies of interest.

This paper focuses on the design of wideband medium gain antennas, involving a few elements only. This type of antennas is becoming more and more popular in the deployment of WLAN networks [8-11]. For wideband antennas, the prime parameters are the mismatch and the gain. The single fitness function can then be formulated as

$$\text{cost function} = \sum_{i=1}^{N} a_i \cdot op_a(S_{11_i} \text{ or } VSWR_i) + \sum_{i=1}^{N} b_i \cdot op_b(Gain_i) \tag{1}$$

where the subscript i refers to the i^{th} frequency and N is the total number of frequencies. The weighting coefficients a_i and b_i can be chosen in such a way that they emphasize a certain frequency band and reduce the importance of other frequency bands. In normal circumstances S_{11}, VSWR, and gain are expressed in a logarithmic, a linear, and a logarithmic scale, respectively. The operators op_a and op_b refer to numerical operations that make the fitness function physically as meaningful as possible, for example truncation and so-called punishing operation. Since the performance of an antenna improves only slightly when S_{11} decreases more and more below -10 dB, truncation may be necessary to dampen the effect of very low S_{11}. The values below -10 dB can be truncated to -10 dB before combining all S_{11} values [12], [13]. This truncation approach can also be employed when VSWR values are used to build up the fitness function [14]. A huge disadvantage of this approach is that it is not clear how to determine the best possible thresholds for S_{11} and VSWR. The punishing operation can be taken advantage of if S_{11} does not satisfy the requirements [15]. In order to give equal weight to reflection coefficient and gain, an exponential operation can be employed for both S_{11} and gain [15].

Moreover, in most cases all the pre-mentioned operators and parameters are usually determined by trial and error, which is time-consuming and highly problem dependent. To the knowledge of the authors, a comparison of the performance of different weighted sum PSO fitness functions has not been done yet for wideband medium-gain antennas.

In general multi-objective optimization problems, it is well known that the main disadvantage of the weighted sum method is that an even distribution of the weights among the objective functions does not always result in an even distribution of the solutions on the Pareto front [16]. Although the Normal Boundary Intersection (NBI) method can provide an even distribution for the solutions on the Pareto front, it needs more fitness evaluations [16], which is not suitable for computationally expensive problems, such as antenna optimizations. However, when the multiple objectives are just the mismatch and gain of the antenna, an even distribution of the solutions on the Pareto front is not preferred, because the realized gain metric (the fourth weighed sum fitness function in section 2.1.4) can determine the best antenna power budget correctly. Consequently, only sum-weighted method approaches are investigated in this paper.

Genetic algorithms, such as the nondominated sorting-based genetic algorithm II (NSGA II) [17], also are not considered because they are not weighted sum based methods, but Pareto-based ranking schemes. The best candidate still needs to be determined by the user.

2. Tested Fitness Functions and PSO Optimizer

2.1 Tested Fitness Functions

Besides the realized gain based fitness function, three other commonly used fitness functions were also employed in the tests. The optimization was performed in two phases. At the end of phase 1, the design requirements are met. Phase 2 further optimizes, in this way improving the target parameters beyond these design parameters. The S_{11}, VSRW and gain values at different frequencies are denoted as $S_{11\,i}$ (in dB), $VSRW_i$ and $Gain_i$ (in dB), where the subscript i indicates different simulated frequency points. N indicates the total number of simulated frequency points. S_{11D} (in dB), $VSWR_D$, $Gain_D$ (in dB) and RG_D (in dB) are the design requirements for S_{11}, VSWR, Gain and realized Gain, respectively. They define the first phase.

2.1.1 Fitness Function Based on the Sum of Weighted S_{11} and Gain [15]

$$\begin{aligned}
&Fitness=\sum_{i=1}^{N}M_i-\sum_{i=1}^{N}R_i\\
&S_i=\max(S11_i,S11_D)\\
&R_i=\begin{cases} S11_D & \forall S11_i\le S11_D\\ -S_i-\left|S11_D+S_i\right|^{K_s} & otherwise\end{cases}\\
&M_i=\begin{cases} Gain_i & \forall Gain_i\ge Gain_D\ \&\ \forall S11_i\le S11_D\\ \min(Gain_i,Gain_D) & otherwise\end{cases}
\end{aligned} \tag{2}$$

The sign $\forall$ indicates that this option is taken as soon as this condition is satisfied at all frequencies. K_s is set to 1 for all tested cases in order to reach an equally weighted sum of reflection coefficient and gain. Both the truncation and punishing operations are employed for S_{11} and only the truncation operation is employed for gain. This S_{11} punishing operation can also be considered as an S_{11} emphasis operation. It is clear that the possible maximum sum of all R_i is $-S_{11D}*N$. It can be achieved if all $S_{11\,i}$ are smaller than S_{11D}. But the possible maximum sum of all M_i is not pre-known. In the first phase, the optimization objective is achieving the design requirements (i.e. both return loss and gain), if the first phase successful, it is able to optimize further to achieve the best possible antenna performance.

2.1.2 Fitness Function Based on the Sum of Weighted VSWR and Gain [14]

$$\begin{aligned}
&Fitness=-(\sum_{i=1}^{N}M_i+\sum_{i=1}^{N}R_i)\\
&S_i=\max(VSWR_i,VSWR_D)\\
&L_i=\max(Gain_D-Gain_i,0)\\
&Condition:\forall VSWR_i\le VSWR_D\ \&\ \forall Gain_i\ge Gain_D\\
&R_i=\begin{cases} VSWR_i & Condition:True\\ S_i & otherwise\end{cases}\\
&M_i=\begin{cases} -Gain_i+Gain_D & Condition:True\\ L_i & otherwise\end{cases}
\end{aligned} \tag{3}$$

This fitness function is an equal weighted sum of M_i and R_i. The truncation operation is employed for both VSWR and gain.

2.1.3 Fitness Function Based on Gain with Return Loss Constraints

$$\begin{aligned}
&Fitness=\sum_{i=1}^{N}M_i+P\\
&Condition:\forall S11_i\le S11_D\ \&\ \forall Gain_i\ge Gain_D\\
&P=\begin{cases} 0 & \forall S11_i\le S11_D\\ C_P & otherwise\end{cases}\\
&M_i=\begin{cases} Gain_i & Condition:True\\ \min(Gain_i,Gain_D) & otherwise\end{cases}
\end{aligned} \tag{4}$$

This fitness function is a sum of M_i plus a constant punishing function P. This punishing function P takes care of the influence of the return loss, where C_P is a well chosen constant value. The return loss punishing function can also be based on the VSWR. The performance of this fitness function will be identical if the constraints of S_{11} and VSWR take the same value of $|\Gamma|$.

2.1.4 Fitness Function Based on Realized Gain [18-20]

$$\begin{aligned}
&Fitness=\sum_{i=1}^{N}Z_i\\
&K_i=a\cdot(S11_i\text{ or }VSWR_i)+Gain_i\\
&Z_i=\begin{cases} K_i & \forall Z_i\ge RG_D\\ \min(K_i,RG_D) & otherwise\end{cases}
\end{aligned} \tag{5}$$

If all realized gains at the simulated frequencies satisfy the equivalent performance requirements, Z_i will take the value of the realized gain K_i. Otherwise, K_i is truncated to RG_D when K_i is larger than RG_D. The fitness function is just a sum of the Z_i.

For simplicity, the fitness functions 1, 2, 3, and 4 are abbreviated as FF1, FF2, FF3, and FF4, respectively. These fitness functions can be used to find the best performing antenna in the required band. If the objective is to achieve the largest possible bandwidth at a centered frequency, the fitness function employed in [21] can be used. It is clear that all the pre-mentioned four fitness functions need to be optimized to a maximal value to achieve the best antenna performance. There are two optimization phases embedded in these fitness functions. The first phase is to achieve the design requirements (for FF1, FF2, and FF3) or equivalent performance requirements (for FF4). Once the first phase is successfully completed, in the second phase the optimizer continues to search for the best performing structure.

In the first phase, all these four fitness functions take into account both return loss and gain, but in the second phase only FF2 and FF4 are capable to further optimize return loss. In the second phase, FF1 and FF3 can be considered the same, but in the first phase, FF1 is supposed to outperform FF3 because the performance of different S_{11} which do not meet the design requirements can be distinct for FF1. Without prior knowledge it is very difficult to set the best possible S_{11} thresholds to jump to the second phase for FF1 to further optimize S_{11}. Also, this is not really necessary because further optimizing S_{11} may not lead to a significant improvement of the realized gain if S_{11} is already smaller than -10 dB (S_{11D} is usually set to be -10 dB).

It is obvious that FF1 emphasizes the return loss more than FF2 and FF4 before achieving the design requirements, due to the punishing operation applied to the return loss. If this punishing operation is removed, the performance of FF1 should be quite similar as the performance of FF2 before achieving the threshold value if the S_{11D} and $VSWR_D$ have identical reflection coefficients. FF4 is expected to converge faster than the others in a PSO optimization, due to the fact that the return loss and gain are not separately checked for the design requirements.

2.2 EM Solver and PSO Optimizer

All EM evaluations were performed with a full wave solver based on the moment of methods, developed for quasi-3D multilayered antenna structures [22], [23]. During the optimization it was assumed that the ground plane and the substrate have infinite dimensions in order to reduce the total number of unknowns.

The equations for updating the velocity and position in the PSO algorithm are:

$$v_n = w v_n + c_1 \mathrm{rand}()\left(p_{best,n} - x_n\right) + c_2 \mathrm{rand}()\left(g_{best,n} - x_n\right) \tag{6}$$

$$x_n = x_n + v_n \tag{7}$$

where v_n is the velocity of the particle in the n^{th} dimension and x_n is the particle's position in the n^{th} dimension. w is referred to as inertia weight. The acceleration coefficients c_1 and c_2 determine the relative 'pull' of the personal best *pbest* and the global best *gbest*. The uniform random number generator rand() injects the unpredictability of the movements of the particles. All these settings of PSO was taken from [24] except for the number of population and the termination conditions.

A piece of memory was employed to record all evaluated antenna configurations and the corresponding fitness values. For already calculated cases the EM evaluation was skipped and the fitness was directly read out from the memory. This reduces considerably the number of calls to the EM solver.

3. 3-element Planar Array

A novel 3-element planar array of microstrip E shaped patches is designed for use in 2.4-2.5 GHz ISM applications. In order to increase the bandwidth of the original prototype [25], the rectangular patches are replaced by E-shaped patches. The topology is sketched in Fig. 1. The antenna is fed by a 50Ohm coaxial cable at the back side. In general, the structure is controlled by 12 parameters. To simplify the design, we choose similar E shapes and set B1=A1*2, B2=A2, B3=A3-2mm, B4=A5, B5=A4 and element B and C are identical. So there are only 7 parameters to be optimized, which makes it a low dimension problem. All parameters can vary with a resolution step of 2 mm except the probe position, which can vary with a resolution step of 0.5 mm.

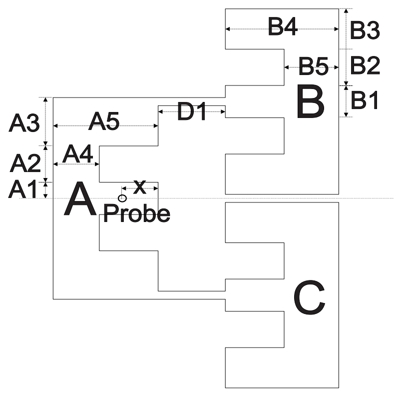


Fig. 1. 3 element array antenna.

The optimization goal is to achieve an S_{11} better than -10 dB in the operating frequency band from 2.25 to 2.55 GHz (12.5 % bandwidth). Inside this band the gain has to be greater than 13 dB, which gives a realized gain RG_D of 12.54 dB. This gives $Gain_D$ = 13 dB, S_{11D} = -10 dB and $VSWR_D$ = 1.9. For cost reasons, a standard FR4 substrate (relative permittivity 4.7, thickness 0.8 mm, tan δ = 0.014) is chosen. A single-layered structure with FR4 is by far not able to deliver the necessary bandwidth. Therefore, the thickness is increased by separating the FR4 substrate from the ground plane by 8 mm high spacers. C_P in FF3 is set to -5 by a trial and error approach. The performance of FF3 is not very sensitive to this value. Five frequency points, i.e. 2.25 GHz, 2.325GHz, 2.40 GHz, 2.475 GHz, and 2.55 GHz, are considered. Both swarm sizes 20 and 30 were tested. The maximal iteration number is 200 for each fitness function. 10 trials were executed for both swarm sizes. In all cases, only a uniform random number generator was used. To further improve the fairness of comparison, the initial seed for the uniform random number generator rand() in (6) for the same trial number for the same swarm size was set to be identical for the four fitness functions. This implies that the initial positions and velocities of the swarm for the same trial number are identical.

Fitness functions FF1, FF2, and FF3 are purely numerically inspired in order to achieve a good antenna power budget. FF4 however is really physically based on the antenna power budget itself. So, it is quite meaningful to check the correctness of FF1, FF2, and FF3 by FF4. The average fitness of the *gbest* for FF1, FF2 and FF3 re-evaluated by FF4 versus the number of iterations, together with the FF4 *gbest*, are plotted in Fig. 2. It is clear that all four fitness functions are effective. Initially, the fitness value of FF2 and FF4 are quite similar and better than for the other two fitness functions. This means that FF2 emulates the power budget condition more precise than FF1 and FF3. In other words, the return loss emphasis operation in FF1 and the constant return loss punishing operation in FF3 have a bad influence on the *gbest* interpretation at the initial state.

It has to be pointed out that Fig. 2 proves the correctness of all four fitness functions. However, it is quite possible that the *gbest* of the fitness functions FF1, FF2, and FF3 are not the candidates which have the best power budget performance for a certain trial. It is also interesting to examine the number of EM calls for all fitness functions according to three criteria: 1. to achieve equivalent performance requirements (i.e. realized gain), 2. to achieve the design requirements (i.e. both return loss and gain), and 3. to converge. All results are summarized in Tab. 1 for swarm sizes 20 and 30. If the optimization didn't achieve the equivalent performance or design requirements, the total EM evaluation number of that trial was used to calculate the mean and standard deviation. It is clearly seen that FF4 outperforms all others. The difference of the last two rows clearly proves that the *gbest* of FF1, FF2 and FF3 are not always the candidates which have the best power budget performance for a certain trial. This means that during the second phase of the optimization, FF1, FF2 and FF3 aren't that correct in power budget performance as fitness function FF4

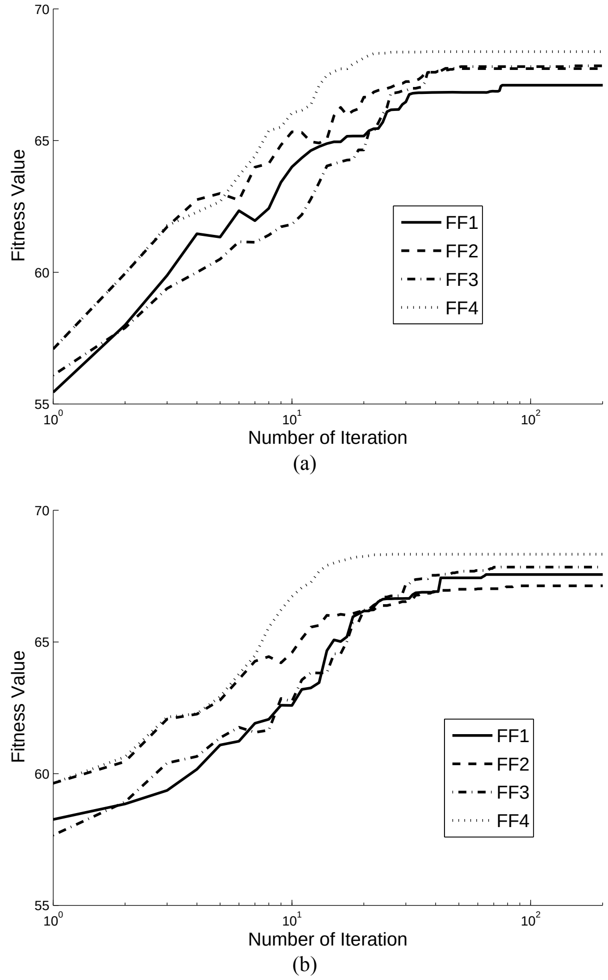
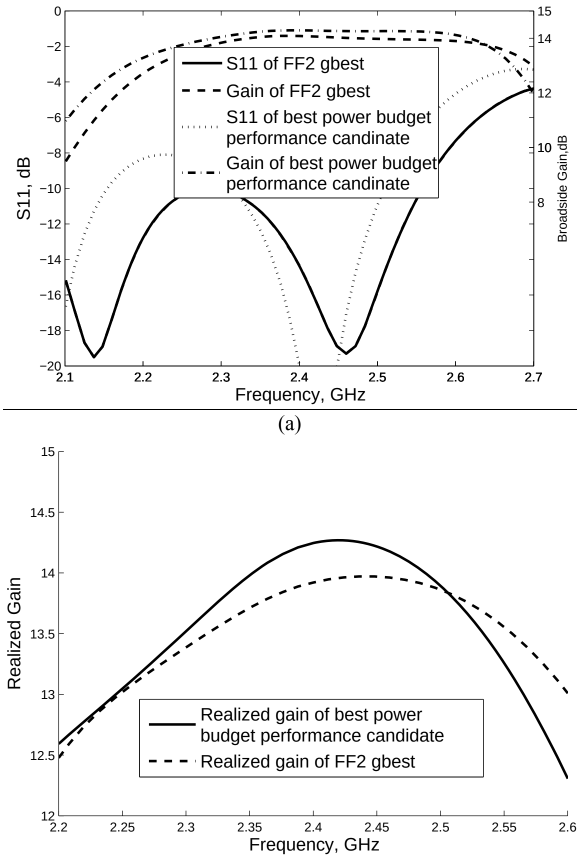


Fig. 2. Average fitness of the FF1, FF2, and FF3 g_{best} re-evalutated by FF4 versus the number of iterations. (a) Swarm size = 20, (b) swarm size = 30.

Swarm size	20				30			
Fitness function	FF1	FF2	FF3	FF4	FF1	FF2	FF3	FF4
Number of Trials Achieving Required Realized Gain	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**
Number of Trials Achieving Design Requirements	**10**	**10**	**10**	9	**10**	**10**	**10**	9
Average Number of EM Evaluations to Achieve Equivalent Performance Requirements (STD)	168 (86)	152 (75)	221 (77)	**151 (64)**	268 (60)	215 (99)	251 (131)	**210 (83)**
Average Number of EM Evaluations to Achieve Design Requirements (STD)	313 (152)	**282 (104)**	395 (121)	282 (122)	536 (215)	**366 (78)**	461 (194)	400 (143)
Average Number of EM Evaluations to Converge (STD)	834 (165)	788 (123)	919 (201)	**568 (62)**	1113 (142)	1237 (203)	1308 (253)	**798 (119)**
Average Fitness of Best Equivalent Performance (STD)	68.04 (1.13)	68.35 (0.07)	68.28 (0.26)	**68.37 (0.06)**	68.23 (0.20)	68.16 (0.33)	68.31 (0.06)	**68.33 (0.08)**
Average Fitness of FF1, FF2 and FF3 *gbest* Re-Evaluated in FF4 (STD)	67.10 (1.24)	67.73 (0.49)	67.83 (0.54)	**68.37 (0.06)**	67.56 (0.63)	67.13 (0.69)	67.84 (0.53)	**68.33 (0.08)**

Tab. 1. The average best fitness and average number of EM evaluations with their standard deviations (STD) over 10 trials for swarm size =20 and 30 for 3 element array antenna.

For each fitness function, the S_{11}, broadside gain, and realized gain of the best power budget performance candidate structure are plotted in Fig. 3, together with the optimal *gbest* of FF2. This is done considering the global batch of 20 trials, consisting of both the 10 trials for swarm size 20 and the 10 trials for swarm size 30. This best power budget performance candidate turns out to be identical for all four fitness functions. The *gbest* of FF2 can be considered as the best power budget performance candidate structure which meets the design requirements (i.e. both return loss and gain). Fig. 3 clearly illuminates that the best power budget performance candidate has a better realized gain than that of the *gbest* of FF2 inside the band 2.25-2.51 GHz, although it does not achieve the design requirements. In the band 2.51-2.55 GHz, the realized gain of the FF2 *gbest* is superior due to the poor impedance matching of the FF4 *gbest*.



(a)

(b)

Fig. 3. S_{11}, broadside gain (a), and realized gain (b), of the best power budget performance candidates for different fitness functions.

4. 4-element Linear Array

A highly compact low-cost and strongly coupled 4 element linear array antenna was also chosen as benchmark antenna. This antenna was first introduced in [26] and recently optimized by both PSO and GA in [24]. The topology of this antenna is shown in Fig. 4. The structure is controlled by 19 parameters, which makes it a high dimension problem. All parameters can vary with a resolution step of 2 mm except the probe position, which can vary with a resolution of 0.5 mm. The optimization goal is to achieve an S_{11} less than -10 dB in the operating frequency range from 3.4 to 3.8 GHz, and inside this band the gain has to be larger than 13 dB. This gain requirement comes from the physical situation in which the antenna is going to be used. This means RG_D = 12.54 dB, $Gain_D$ = 13 dB, S_{11D} = -10 dB and $VSWR_D$ = 1.9. Five equidistant frequency points 3.4 GHz, 3.5 GHz, 3.6 GHz, 3.7 GHz, and 3.8 GHz are considered to be evaluated. C_P in FF3 was also set to 5.

The thresholds for S_{11} at different frequencies in the fitness function used in [24] were carefully tuned by the multi-step PSO optimization with small swarm size. This was not done in this paper.

In order to find the best performance candidate array, the maximal iteration number value was set to 1000 for both swarm size 60 and 100. Another termination condition was that the maximal number of EM evaluations is 12000. This corresponds to a non-stop simulation time of about 10-days on an Intel(R) I7 CPU @ 2.8 GHz workstation with 4 GB memory. 10 trials were executed for each fitness function for both swarm sizes. In all cases, the initial seed of the employed uniform random number generator was also set to be identical for the same trial number.

The average fitness of the *gbest* for FF1, FF2 and FF3 re-evaluated with FF4 versus the number of iterations, together with the FF4 *gbest*, are plotted in Fig. 5, which clearly proves again 1) the correctness of all four fitness functions, 2) that FF2 emulates the power budget condition more precise than FF1 and FF3 at the initial state. All interesting results are summarized in Tab. 2 for swarm sizes 60 and 100. Again, it is clearly seen that FF4 outperforms all others. For each fitness function, the S11, broadside gain, and realized gain of the best power budget performance candidates, together with the simulation results of the prototype of [24] are plotted in Fig. 6. All dimensions are summarized in Tab. 3, according to the definitions given in Fig. 4. L and W are the total length and the widest width of the candidate array, respectively. FF1 and FF3 achieve an identical power budget performance candidate structure, and FF2 and FF4 achieve another identical structure. All the plotted candidates meet the design requirements. It is clear that the realized gain has been improved considerably compared to the prototype described in [24]. However, it has to be emphasized that these candidates are about 20 % longer and 30 % wider than the prototype of [24]. All the found arrays have identical parameters except the dimension C6.

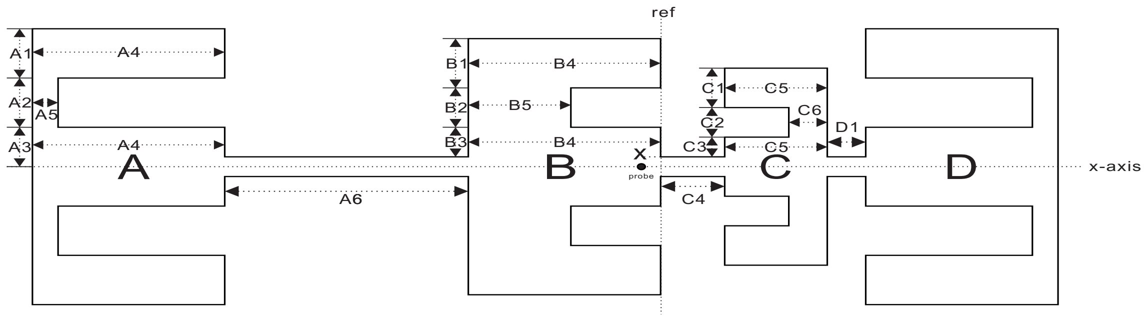


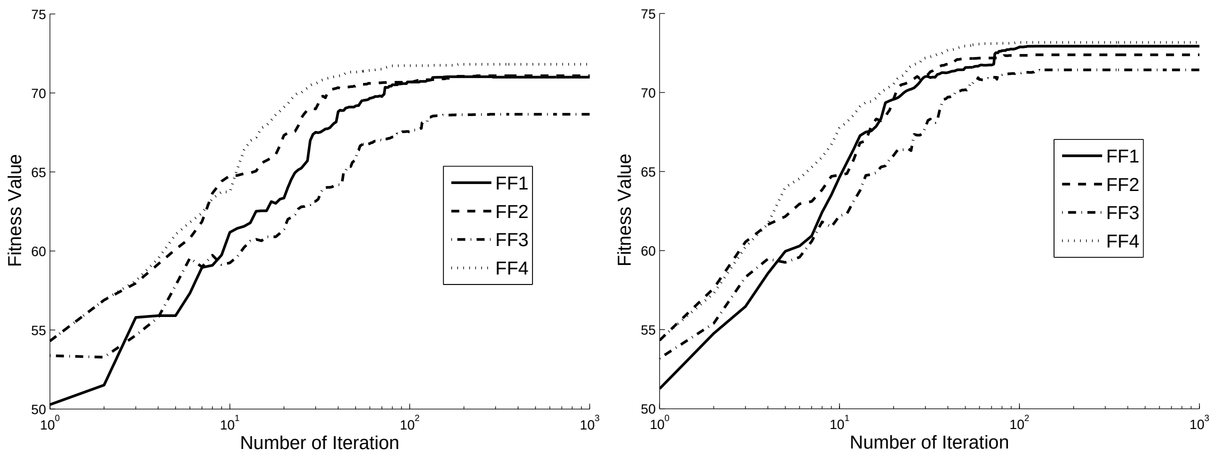
Fig. 4. Antenna array prototype top view.

Swarm size	60				100			
Fitness function	FF1	FF2	FF3	FF4	FF1	FF2	FF3	FF4
Number of Trials Achieving Required Realized Gain	**10**	**10**	9	**10**	**10**	**10**	**10**	**10**
Number of Trials Achieving Design Requirements	**10**	**10**	7	**10**	**10**	**10**	**10**	**10**
Average Number of EM Evaluations to Achieve Equivalent Performance Requirements (STD)	1160 (1062)	798 (454)	2790 (4188)	**731 (337)**	935 (785)	933 (632)	1650 (1328)	**835 (593)**
Average Number of EM Evaluations to Achieve Design Requirements (STD)	2412 (2005)	**1397 (925)**	4879 (4510)	1434 (1020)	2216 (2157)	1652 (782)	2760 (1984)	**1629 (1423)**
Average Number of EM Evaluations to Converge (STD)	7503 (2714)	5872 (2804)	9464 (2482)	**4854 (1735)**	9559 (2500)	7895 (2731)	10072 (2875)	**7763 (3169)**
Average Fitness of Best Equivalent Performance (STD)	71.28 (2.73)	71.31 (3.10)	69.85 (4.16)	**71.85 (3.03)**	73.12 (2.61)	72.51 (2.33)	71.62 (3.54)	**73.17 (2.47)**
Average Fitness of FF1, FF2 and FF3 *gbest* Re-Evaluated in FF4 (STD)	71 (3.03)	71.08 (3.37)	68.65 (5.68)	**71.85 (3.03)**	72.92 (2.71)	72.38 (2.46)	71.42 (3.51)	**73.17 (2.47)**

Tab. 2. The average best fitness and average number of EM evaluations with their standard deviations (STD) over 10 trials for swarm size =60 and 100 for 4 element array antenna.

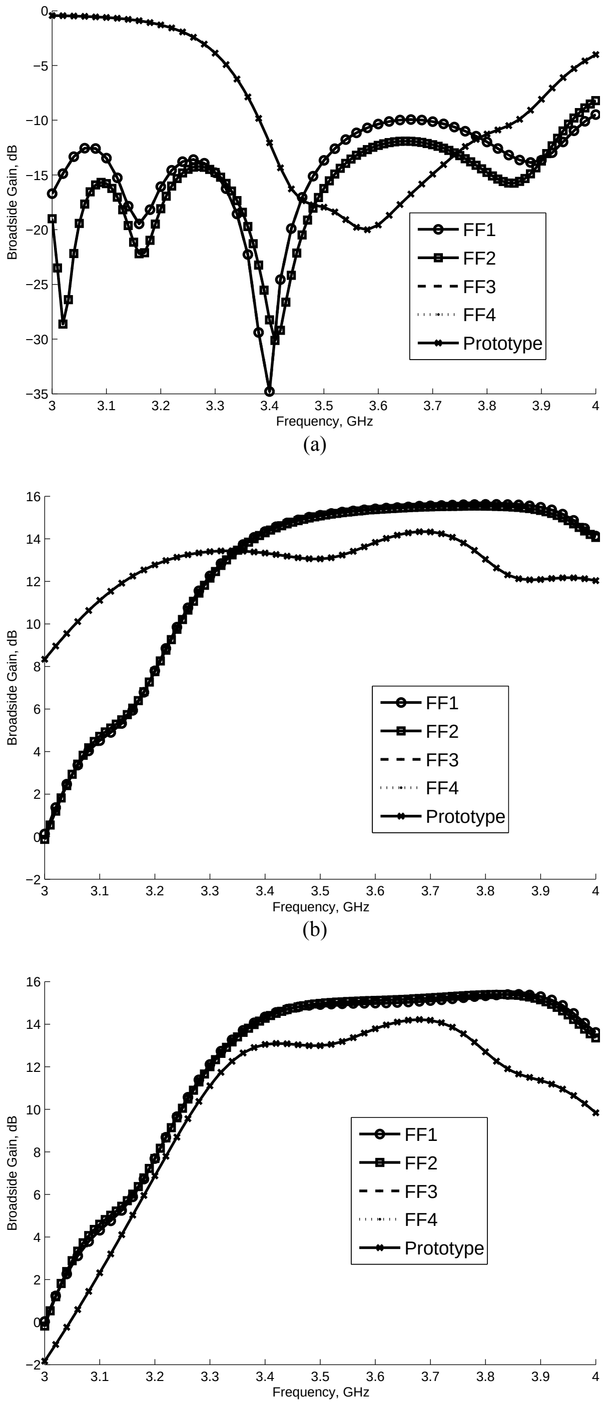
Fitness Function	A1	A2	A3	A4	A5	A6	B1	B2	B3	B4	B5	C1	C2	C3	C4	C5	C6	D1	X	L	W
FF1	12	12	12	30	4	28	18	10	8	28	16	12	4	12	24	30	**12**	24	5	194	72
FF2	12	12	12	30	4	28	18	10	8	28	16	12	4	12	24	30	**10**	24	5	194	72
FF3	12	12	12	30	4	28	18	10	8	28	16	12	4	12	24	30	**12**	24	5	194	72
FF4	12	12	12	30	4	28	18	10	8	28	16	12	4	12	24	30	**10**	24	5	194	72
Prototype [24]	10	10	8	30	4	38	10	8	6	30	16	8	6	4	10	16	6	6	5	160	56

Tab. 3. Dimensions of the best power budget candidate arrays found by different fitness functions and the original prototype (unit: mm).



(a) (b)

Fig. 5. Average fitness of the FF1, FF2, and FF3 *gbest* re-evalutated by FF4 versus the number of iterations. (a) Swarm size = 60, (b) swarm size = 100.



(a) (b) (c)

Fig. 6. S_{11} (a), broadside gain (b), and realized gain (c) of the best power budget performance candidates for different fitness functions.

5. Conclusion

In the design of wideband medium gain arrays our advice is to use the fitness function based on realized gain. Among the four commonly used PSO fitness functions tested, it is slightly faster while reaching a similar global best candidate structure.

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